Pergamon

Int. J. Heat Mass Transfer. Vol. 37, Suppl. 1, pp. 425–431, 1994

0017-9310/94 $6.00+0.00

0017-9310(93)E0050-Q

Turbulent transition and maintenance of turbulence; implication to heat transfer augmentation

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Abstract—In this communication we present formulation of a criterion for (turbulent) transition for wall bounded flows. In addition, we discuss the implication of the findings to heat transfer augmentation. The presentation has three parts. First, we argue that transition to turbulence should be interpreted solely in the context of the system's capacity to maintain turbulent motions. Second, we propose a model that links maintenance of turbulence to the intermittent transfer of vorticity from the wall region (the demarcation of transition follows directly from the model: it is simply a limiting case of the conditions for maintenance). Third, we apply the findings to estimate heat transfer in the transition region. Experimental results show a good agreement with the proposed criterion; they show a strong cause–effect coherency between vorticity production and the system capacity to maintain turbulence (a correspondence implied by the proposed model). It was also found that the heat transfer coefficient in the transition region is significantly increased when the transition Reynolds number is lowered through deployment of suitable augmentation schemes.

INTRODUCTION

OUR APPROACH rests on the following argument. In general, a breakdown of a laminar flow (or any bifurcation) is possible only because the system (under the existing boundary conditions) is capable to support the new emerging state (or states).‡ When the new state is known, then, in principle, the conditions for its maintenance are also determinable; the latter, when found, can be used to establish the boundary of the parametric domain, outside which the new state could not exist. In our interpretation this boundary defines the demarcation of transition to the new state; in the case of turbulent transition, it refers to the system's latency for turbulence, not its imminent manifestation. With this definition, the question of transition is decoupled from the causes and details of the breakdown of laminar flow, it depends only on the intrinsic features of the turbulent state; because of this self-contained aspect of the formulation, it should consistently yield a reproducible and predictable demarcation.§ On a more general level, this approach explicitly communicates a larger and an obvious point: *turbulence exists because it is self-sustaining and not because its causal platform—a laminar basic state—is necessarily self-destabilizing.*

The following simple (and familiar) example illustrates some key points. For flows in a straight tube, turbulence, once present, can sustain itself for Reynolds numbers higher than about 2300 (based on the mean velocity and the tube diameter). As a result, in common practicc, $Re \approx 2300$ is typically assigned as the 'point' that separates the two flow regimes. Attempts to demarcate transition approaching it from the laminar side, on the other hand, lead to difficulties: for this geometry, laminar flow could exist well beyond the above value of Re number: its breakdown would depend on the external factors.‖ This example shows that transition is already marked in terms of the system's latency for turbulence, and, even more significantly, this choice was the only coherent alternative.

The above suggests that a comprehensive criterion for transition (together with the answer to the adjoining question why there is turbulence) should be sought and formulated through the explication of self-sustaining features of turbulent motions, rather than

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‡This is of course a truism, but nevertheless still true.

§Accessing transition from the laminar side, one finds that the breakdown of a laminar flow would frequently depend on the factors extrinsic to the laminar basic state (for instance, the type and intensity of external perturbations, the degree of roughness at the confining walls, etc.); these states are called 'conditionally stable' states, ref. [1].

‖In the original experiments of O. Reynolds in 1883, transition to turbulence occurred around $Re \approx 13\,000$, see ref. [2]. Significantly, and in accord with the above, for this case, any algorithmic compression of the mathematical representation of the laminar flow (e.g. stability analysis applied to the basic laminar state), would fail to predict transition that could match the experimental findings. This reveals the difficulties of using breakdown of laminar flow as a consistent marker of transition.

NOMENCLATURE

a, b, d, d_1	geometric parameters of eddy promoters (Fig. 3)
c	height of isosceles triangle (Fig. 1)
C_1, C_2	constants
Fo	Fourier number (v_t/r_o)
H	channel height (Fig. 2)
J_0	Bessel function of the first kind of order zero
l_1, l_2	spacing between eddy promoters (Fig. 2)
L	periodicity length
Nu	Nusselt number (characteristic length is channel height)
Δ_p	fluctuating pressure
Re	flow Reynolds number (characteristic length: diameter for the tube, and channel height for channel flow)
Re_τ	shear Reynolds number ($u_\tau \Lambda/v$)
r_o	radius of a vortex
t_c	convective time scale
t_v	dissipation time scale
u_τ	friction velocity
V	velocity
v'	fluctuating velocity in y-direction
W	channel width (Fig. 2)
x	streamwise coordinate
y	normal coordinate
z	spanwise coordinate.

Greek symbols

δ	viscous layer thickness
ζ	distance from the apex (Fig. 1)
Λ	integral length scale
λ_n	roots of $J_0(\lambda) = 0$
μ	dynamic viscosity
v	kinematic viscosity
ρ	density
τ	temporally averaged shear stress
$\bar{\omega}$	mean vorticity of the vortex
ω_i	initial vorticity of the vortex.

Subscripts

c	convective
max	maximum
p	pressure eddies at the wall
trans	at the transition to turbulence
w	wall
v	dissipation
∞	free-stream.

Superscript

+	wall coordinate.

via details of the system-specific, and frequently finite-perturbation-dependent, breakdown of laminar flows.†

THE MECHANISM AND MODEL

The mechanism

The basic elements, which constitute the essential structure of the model for maintenance, readily precipitate from the existing evidence. First, on the simplest level, turbulence can be perceived as a three-dimensional interaction between vortex filaments ('eddies' of different size). Second, each of these eddies is comprised of a fluid element formed by rolled-up vorticity. Third, for wall bounded flows, the positive production of vorticity takes place in the wall region. Taken together, they imply that in wall bounded flows the maintenance of turbulence requires intermittent transfer ('fueling') of vorticity from the wall layer.‡

The vorticity transfer from the wall layer, as interpreted by the proposed mechanism, is based on interdependency between the 3D perturbation pressure field (and the accompanying velocity fluctuations) and the wall viscous layer separation. In simplest terms, the mechanism is this: local turbulent pressure fluctuations at the wall (related globally to the perturbation velocity field, including the far field) will intermittently destabilize the wall viscous layer (by means of its adverse pressure gradient sweeps), thus inducing a local vorticity roll-up.§ Subsequent interaction of the rolled-up vortex with the adjacent velocity and associated pressure field leads to the vortex lift-off, its stretching, and formation of hair-pin structures. Intermittent events of this kind would supply vorticity to the core, thus maintaining conditions for

†The existence of a general self-contained internal mechanism that alone can be credited for the viscous layer destabilization is not supported either by the experimental evidence or by any plausible theoretical argument. Also, on the phenomenological grounds, the nature of the phenomenon whose existence is verified must be sought within the intrinsic features of the phenomenon itself rather than outside it.

‡One might recall here that as early as 1915, G. I. Taylor [3] suggested that the dynamic of turbulent motions should be regarded as an effect of diffusion of vorticity rather than momentum.

§It is interesting to note that the effect of fluctuation pressure on the viscous layer was discussed, in a somewhat different context, by G. I. Taylor [4]. He writes: "It seems that the way in which turbulence is most likely to affect the boundary layer is through the action of local pressure gradients which necessarily accompany turbulent flow. If these are sufficiently great, for instance, and directed oppositely to the flow, a reversal of flow at the surface, or separation, will occur". Offen and Kline [5] have also clearly recognized the role of transient adverse pressure gradient on the viscous 'sub-boundary layer' behavior.

subsequent destabilization of the wall layer (which, in turn, serves as a continuous source of vorticity). The proposed mechanism is quite consistent with existing knowledge in the field as the following several chosen examples show.

In turbulent flows the field of velocity perturbations (produced by interaction of vortex filaments) is always accompanied with the attendant fluctuating pressure field; for incompressible flows, the two are related through Poisson's equation (which is obtained as the divergence of the momentum equation, e.g. ref. [6]). The latter indicates that the perturbation pressure at a point is created by contributions from velocity fluctuations of the entire flow field. This means, first, that the local pressure fluctuations are not strongly correlated with the local velocity perturbation, and second, that there exist much higher correlation of fluctuating pressure between two different points in the field,† than that of the perturbation velocities. In accord with the above, Kim in a numerical simulation of flow between parallel plates at $Re_\tau = 180$ [8] has shown that the iso-correlation contours for pressure did not exhibit any presence of inclined structures (that would reflect hair-pin vortexes in the wall region), whereas in the contours of streamwise velocity fluctuation, they were very much evident: the maximum correlation of pressure fluctuations was aligned at 90° to the wall plane, and that for streamwise perturbation velocity at 20°. Kim's results also showed that the two-point correlation for fluctuating pressure between $y^+ = 50$ and $y^+ = 0$ in the normal direction was about 50%; and between the mid-channel and the wall was approximately 15–20%. This suggests that there is a strong communication between the turbulent core and the wall layer via a shared pressure field.

Measurements of the convective velocity of pressure eddies at the wall further reinforce the notion that the effective center of the source term of the pressure fluctuations at the wall is moving outside the viscous layer. For instance, Willmarth and Wooldridge [9], for a thick turbulent boundary layer, reported that the convective velocity of the pressure eddies at the wall (V_p) was in the range of 0.56–0.83 times the free-stream velocity, V_∞ (or, for the flow parameters of the experiments, $V_p = 17.5$–25.9 of u_τ); based on the mean velocity profile, the position corresponding to these velocities‡ was from $y^+ \approx 120$–3000 (with $\delta^+ \approx 16\,500$ at the location where the data were taken). Similar results were obtained by other investigators: Bull and Willis, as reported in ref. [9], measured the convective speed in the range from 0.7 V_∞ to 0.85 V_∞; Kim [8] reported $V_p = 0.72\ V_{max}$ (and $V_p = 13u_\tau$), where V_{max} is the mean mid-channel velocity. In all the cases, the highest convective speed goes with the low-frequency pressure fluctuations (that is, large eddies).

The presence of intermittent flow separations at the wall, a *condicio sine qua non* for maintenance of turbulence, implies the presence of the wall shear stress fluctuation with the amplitude of the order of the mean wall shear stress, τ_w. If the pressure fluctuations were to induce separation, the amplitude of pressure fluctuations at the wall, in non-accelerating flows, must be also of the order of (but larger than) τ_w, ref. [10].§ The available experimental evidence is consistent with this inference. For instance, Willmarth and Wooldridge [9] reported that the root-mean-square wall pressure was 2.19 times the wall shear stress; Bull and Willis found for the same to be $2.7\tau_w$.

Lastly, once separated, the secondary instability of the three-dimensional pressure and velocity field will lead to the vortex stretching, its lift-off, and formation of hair-pin structures. Based on flow visualization, a plausible model for vortex stretching and its lift-off was proposed by Offen and Kline [5].

The model

Expanding on the above, we propose a simple model for maintenance of turbulence.‖ Congruous with the established picture, we base our formulation of the criterion for maintenance on the motions associated with the process of vorticitiy transfer from the wall layer. To this end, we argue that for turbulence to be sustained, that is, for vorticity to be convected to the outer region without being dissipated during the transfer process, the time scale associated with this convection, t_c, must be smaller than the dissipation time, t_v, both expressed in terms of the viscous layer thickness δ. Specifically, with $t_c = C_1\ \delta/v'$, and $t_v = C_2\ \delta^2/\nu$, for maintenance: $t_c/t_v \geqslant 1$, and $\delta v'/\nu \geqslant$ a constant $(=C_1/C_2)$. In the above, v' is a characteristic intensity of the perturbation motion responsible for vorticity transfer, C_1 is a constant of order of unity, and $1/C_2$—based on diffusion con-

†Heisenberg [7] and Bachelor [6] have shown that for a field of isotropic turbulence, the two-point correlation for pressure is related to the fourth moment of the velocity fluctuations.

‡Interpretation of the measured velocity of the source should be done with some care: the source velocity need not be that of the mean velocity corresponding to the source actual position (it could differ from the mean velocity of its location for the value of the perturbation component of the streamwise velocity; a small departure from the mean value, if ignored, could lead to a significant error in the computation of the source position because of the semi-logarithmic relation between the two).

§The above inference is based on a first-order approximation of the momentum balance which considers only friction at the wall as the primary contributor to the pressure drop; a comparison between the pressure drop for a separated region and one without separation readily leads to the above estimate of pressure fluctuations necessary for a local viscous layer roll-up.

‖The concept was first formulated by one of the authors of this communication and partly described in refs. [10–12].

trolled dissipation of vorticity†—is of the order of 50 or so. Since in turbulent flows the root-mean-square of v' scales with the 'friction velocity' u_τ $(=(\tau_w/\rho)^{1/2})$, the condition for maintenance reduces to:

$$\frac{\delta \cdot u_\tau}{\nu} \geqslant B \tag{1}$$

where $B \approx C_1/C_2$ is a constant whose value is around 50. The left-hand side in the above inequality is the maximum thickness of the viscous layer expressed in the wall units, δ^+.

This result should be interpreted as follows. In a fully developed turbulent flow, the viscous wall layer is confined to the region $\delta^+ \leqslant B$. In other words, the viscous layer will intermittently grow, not unlike the growth of laminar boundary layer at the leading edge of a flat plate, approaching its limiting value $\delta^+ = B$, before its break-up induced by adverse pressure gradient sweeps.

In the context of our definition of transition, which is based on the system's capacity to maintain turbulence, one can readily demarcate transition from the above criterion by simply recognizing that when δ^+ reaches the integral length scale of the system (Λ), turbulent motions cannot be self-sustaining any more. Thus, the criterion for system latency for turbulence (in our interpretation—turbulent transition) can be expressed as

$$Re_\tau = \frac{\Lambda \cdot u_\tau}{\nu} \geqslant B. \tag{2}$$

The value of Λ for flows between parallel plates is approximately channel half-height; for a pipe flow, Λ is of the order the pipe radius; for a flow over a flat plate, Λ is of the order of the boundary layer thickness.

The interpretation of the above inequality is this: for $Re_\tau < B$, turbulence cannot sustain itself; the criterion, however, does not imply that for $Re_\tau > B$ the flow would be necessarily turbulent (that is, the flow could be 'conditionally stable'). For non-accelerating flows, B, as discussed above, is of the order of several tens.

EXPERIMENTAL RESULTS

Transition to turbulence

The available experimental results are consistent with the proposed concept. For instance, the model implies that the growth of the viscous layer in fully developed turbulent flow is limited to δ^+ of 40–60; this is in agreement with observations, e.g. refs. [13, 14]. Also, the results for several standard geometries, including flow in tubes, between parallel plates, falling films, flow over flat plates, give the value of Re_τ at transition in the expected range.‡

The measurements reported by Eckert and Irwine for the onset of turbulence in a fully developed flow in a triangular duct [15], offer a good test for the model. The cross section of the duct was an isosceles triangle with one sharp angle (12°). In this case both the local wall shear stress and the integral length scale vary circumferentially along the channel (triangle) perimeter. Figure 1 shows the Reynolds number (based on the hydraulic radius) which is necessary to sustain turbulence at different location ($\zeta/c = 0$ is located at the apex). As expected, turbulence appears first at the center of gravity of the triangle (the position where the integral length scale is the largest), and moves with increasing flow rate toward the apex. The prediction, using $Re_\tau = 50$, agrees remarkably well with the reported results.§

The proposed mechanism can be further interrogated by designing experiments in which a modification of the basic system could lead to the enhanced vorticity generation without necessarily increasing the flow rate, and at the same time keeping the integral length scale of the system invariant. The results from such an investigation should demonstrate the connection between the enhanced vorticity production and the system's capacity to sustain turbulence, and provide an additional assessment of the adequacy of Re_τ as a transition marker. With this as the strategy, we have performed a series of experiments, including both two- and three-dimensional geometries.

† A simple model based on dissipation (diffusion to the surrounding fluid) of the rolled-up vorticity, which assumes cylindrical shape of the vortex of radius (r_o), yields for the ratio between the mean vorticity within the vortex, $\bar{\omega}$, and its initial value, ω_i, the following,

$$\bar{\omega}/\omega_i = 4\sum_1^{\infty} \exp(-\lambda_n^2 Fo)/\lambda_n^2,$$

where λ_n are roots of $J_0(\lambda) = 0$, and $Fo = \nu t/r_o^2$. From the above one can estimate C_2. For example, 90% dissipation of the original vorticity content (i.e. $\bar{\omega}/\omega_i = 0.1$) yields $Fo = 1/3$. With this value one gets $t_v = (1/3)(r_o/\delta)^2\delta^2/\nu$, and further, $1/C_2 = (\delta^2/\nu\, t_v) = 3(\delta/r_o)^2$. A plausible range for δ/r_o (the ratio between the maximum size of the viscous layer thickness and the effective radius of the vortex, which undergoes stretching, and therefore reduction in its radius) is estimated to be somewhere between 3 and 5. Using, for instance, the value of 4 yields $1/C_2 = 48$. Other ways to estimate $1/C_2$ lead to a similar result: $1/C_2 = 40$–60.

‡ For example, for flow over flat plate, using $Re_{trans} = (V_\infty x/\nu) = 3.2\times10^5$, evaluating the wall shear stress from the Blazius solution, and taking δ at 5% velocity deficiency, gives $Re_\tau = 51$; for pipe flow, $Re_{trans} = 2300$, and $\Lambda = 0.7r_o$ (reflecting the constraints of pipe geometry), yields $Re_\tau = 47$. Whereas the above choices for values of Λ were to some extent arbitrary, in the range of plausible choices, the values for Re_τ would differ from the above results not more than 20%.

§ The wall shear stress was evaluated numerically [16]; it could be also calculated from the laminar solution of the problem [17]; the integral length scale for each predicted point was taken to be the distance from the point in question, on the line bisecting the sharp angle, to the nearest point at the wall. Only several central points were calculated this way, since the appearance of turbulence in the central part of the channel will affect the wall shear stress everywhere (due to the flow redistribution), and the local shear stress based on the laminar solution becomes inaccurate.

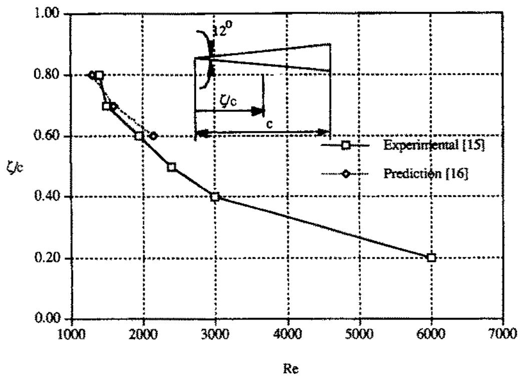

FIG. 1. Transition to turbulence in a triangular duct; experiments [15], and prediction [16].

A high aspect ratio (8.63 : 1) channel has been chosen as a basic geometry. The enhancement of vorticity production, and the resulting early transition to turbulence, was achieved through flow destabilization due to the presence of periodically placed two-dimensional (2D) and three-dimensional (3D) eddy promoters. Figure 2 shows basic geometry and the placement of eddy promoters. One periodicity length is $L = l_1 + l_2$. Channel height $H = 2.54$ cm. Channel width $W = 22.63$ cm. Figure 3 shows a sketch of eddy promoters used in experiments. Geometric parameters of all the investigated cases are presented in Table 1. 3D2D was treated as a three-dimensional augmentation geometry. Variation of geometric parameters controlled the onset of transition to turbulence. A more detailed description of the experimental apparatus can be found in ref. [16]. All experiments were performed in the region where flow can be considered periodically fully developed, as defined in ref. [18]. For the cases in Table 1, $\Lambda \approx H/2$. The measurements of pertinent parameters were carried out using standard methods: velocity profile was obtained from Pitot tube output; the local wall shear stress was recorded by a fence-type shear probe (calibrated against the pressure drop for plane channel flow, and reconfirmed using comparison with the numerical simulation for a system with eddy-promoters on the laminar side of the transition). Flow Reynolds number was based on the average velocity and the channel height. A qualitative analysis of the spectra in the x-direction was used to determine the transition 'point'. In the evaluation of Re_τ at transition, we used for τ_w the spatial average of the wall shear stress over one periodicity length, measured at $z = w/2$, and $y/H = 0$, c.f. Fig. 3. The onset of turbulence was determined via analysis of frequency spectra of the fluctuating component of streamwise velocity: the appearance of a broad-band energy spectrum was used as an indicator of transition. For a number of cases these find-

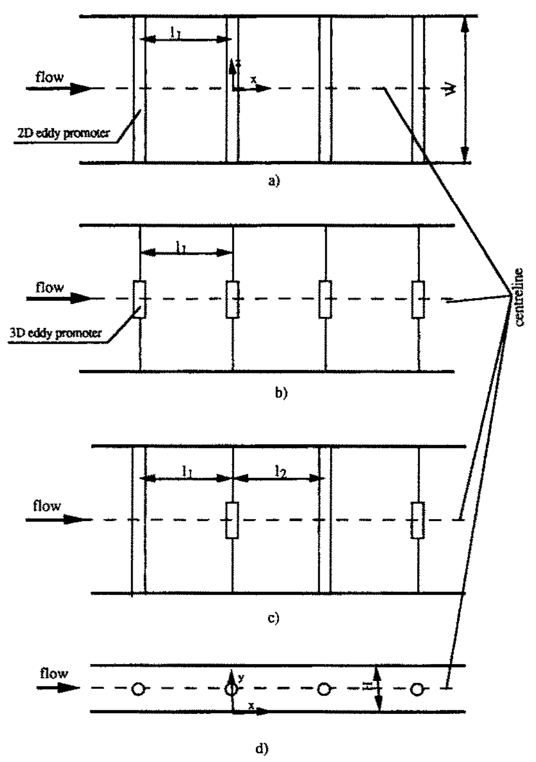

FIG. 2. (a), (b), (c) Schematic of the top view of the test section for: 2D geometries, 3D geometries and 3D2D cases, respectively, and (d) the side view of the test section.

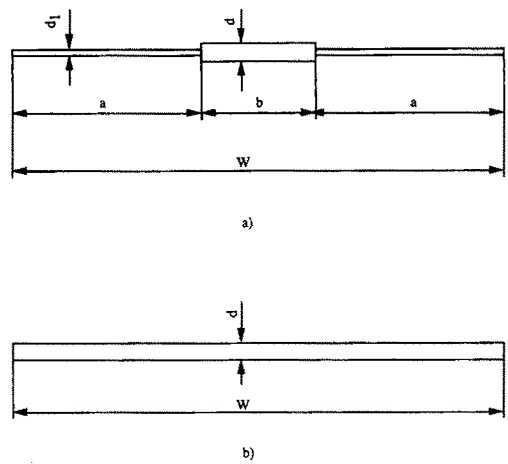

FIG. 3. Schematic view of: (a) 3D eddy promoters and (b) 2D eddy promoters.

Table 1. Geometric parameters of the investigated configurations

Case	Geometry	l_1/H	l_2/H	d/H	d_1/H
825C	2D	7.76	N/A	0.196	N/A
425C	2D	3.88	N/A	0.196	N/A
225C	2D	1.94	N/A	0.196	N/A
125C	2D	0.97	N/A	0.196	N/A
835C	2D	7.76	N/A	0.309	N/A
435C	2D	3.88	N/A	0.309	N/A
3D1	3D	7.76	N/A	0.309	0.125
3D2	3D	3.88	N/A	0.309	0.125
3D2D	3D	3.88	3.88	0.309	0.125

$a/H = 3.315$; $b/H = 2$; $W/H = 8.63$.

Table 2. Measured values for Re_{trans} and Re_τ

Case	Re_{trans}	Re_τ
Plane channel [16]	1465–1584	49–52
825C [16]	880–988	49–53
425C [16]	698–815	46–51
225C [16]	610–695	47–51
125C [16]	574–615	44–48
835C [16]	560–688	43–49
435C [16]	380–430	41–46
3D1 [19]	650–750	41–45
3D2 [19]	550–620	38–41
3D2D [19]	400–500	35–40

ings were further reconfirmed by the changes in slope of the wall shear stress vs flow rate plot, see ref. [16].

Table 2 presents the summary of the results. The presence of eddy promoters displaces the onset of turbulence to lower flow Reynolds numbers [16, 19]: the value of Reynolds number at transition decreased from 1500 for the plane channel, to approximately 400 for the least-stable configuration with eddy promoters. The range of values for Re_τ at transition conforms with the proposed criterion, as can be seen from Table 2: Re_τ at transition was around 40. There is, however, a slight but systematic reduction of its values for the cases where the number of promoters per unit channel length was high, and/or their diameter was approaching the integral length scale of the system; the same trend is seen also in the runs involving 3D geometry. This behavior is perhaps attributable to the fact that here we might have a significant vorticity generation (at the system's integral scale) whose effects on the system's capacity to support turbulence are not entirely captured by the value of the mean wall shear stress (which is used in the model to represent perturbation velocitiy responsible for vorticity transport).

Heat transfer in the transition region

The augmentation technique which was used here, characterized by enhanced vorticity generation, points to some interesting connections between the phenomenon of early transition and heat transfer. In particular, the lower flow rate at a constant wall shear stress (constant Re_τ) at transition, could also mean—provided that the analogy between momentum and heat transfer (Colburn analogy) holds in this region—higher heat transfer coefficients at a lower transition Reynolds number. Recent numerical results, ref. [20], for the case of flow in channels with eddy promoters, show that in the vicinity of the transition region, approaching it from the laminar side, the analogy is indeed valid.

To quantify the implication of the invariance of Re_τ at transition on heat transfer, we recast the Colburn analogy in the following form:

$$Nu \approx 4\frac{Re_\tau^2}{Re} Pr^{1/3}. \tag{3}$$

Because Re_τ is approximately constant for all system at transition, we can infer from the above that if the analogy holds, the transition Nusselt number (Nu_{trans}) should be inversely proportional to the transition Reynolds number, that is

$$Nu_{trans} \approx \frac{\text{const.}}{Re_{trans}}. \tag{4}$$

This means that using augmentation techniques based on enhanced vorticity generation, one would be able to get not only transition at lower flow rates, but also in this region higher heat transfer coefficients than those one can achieve from the corresponding non-augmented systems (at their respective transition Reynolds numbers). We have tested the above proposition on two geometries with eddy promoters. The initial results are given in Table 3. A close agreement between the observed Nu_{trans} and the predicted one, using in equation (3) the measured values for Re_τ and Re at transition (listed in Table 2), is evident. For purposes of comparison, the Nu_{trans} for plane channel is also included in the table.

CLOSING REMARKS

The intent of the communication was to define and formulate a criterion for turbulent transition in the context of the system's ability to maintain turbulent motions. A further objective was to examine the implications of the findings on heat transfer in the transition region. The criterion was derived from a simple concept for self-preserving destabilization of the viscous layer. The concept alone, although based on the already well established principles, of course, does not prove anything. It simply provides an appropriate background where the pieces of the existing knowledge can be put together, allowing for a coherent 'large picture' to emerge. Its ultimate acceptance would depend solely on how well it can do this job, and not on any rigorous mathematical proof of it, for there cannot be any.†

Table 3. Observed and predicted values for Nu_{trans} for two 2D cases

Case	Nu_{trans} (observed)	Nu_{trans} (predicted)
Plane channel ($Re = 1550$)	Not available	5.67‡
825C ($Re = 880$–1006) [19]	10–10.48	9.6–9.91
425C ($Re = 740$) [21]	11	10.96

‡This value is obtained using Petukhov correlation, [22].

†To establish hierarchical causalities within the rich and diverse morphology of the turbulent motions, one cannot rely solely on the formal method, that is, on the conservation principles alone; this simply would not be sufficient. One must assist the process of explication through a concept formulation based on interpretation of the observed events.

The central idea behind the proposed model is that turbulent motions (which are essentially three-dimensional interaction between multiple-scale eddies) are driven by the fluid vorticity. Further, to maintain these motions, the system must have the capacity to intermittently fuel the core with high vorticitiy fluid from the wall layer. Built on this central idea, a sequence of the connecting arguments leads directly to the criterion for maintenance of turbulence, and by extension, to demarcation of transition. It says, that for a system to be able to support turbulence, Re_τ based on the system's integral scale, must be larger than a number of around 40–60. The experimental results are consistent with the proposed criterion: in a series of experiments in which transition Reynolds number was varying for a factor of almost 4, the value Re_τ at transition remained approximately invariant. The experiments had also clearly established a strong correlation between vorticity generation and the system's capacity to support turbulent motions, thus confirming the main proposition of the concept.

Finally, the constancy of Re_τ at transition, together with the analogy between momentum and heat transfer, implies that $Nu_{\text{trans}} \sim 1/Re_{\text{trans}}$. The initial results support this implication: reduction of Re_{trans}, which was achieved with enhanced vorticity production, lead to significant increases in Nu_{trans}.

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